4th SEMI-ANNUAL REPORT

NASA RESEARCH GRANT NGR-33-019-058

INVESTIGATION OF COMPUTER-AIDED CIRCUIT DESIGN

PART I

SYNTHESIS OF DISTRIBUTED SYSTEMS

PART II

SIGNAL FLOW GRAPH APPROACH TO COMPUTER-AIDED DESIGN

September 1, 1966 - February 28, 1967

THE UNIVERSITY OF ROCHESTER
COLLEGE OF ENGINEERING AND APPLIED SCIENCE
DEPARTMENT OF ELECTRICAL ENGINEERING
ROCHESTER, NEW YORK

SEMI-ANNUAL REPORT

NASA RESEARCH GRANT NGR-33-019-058

(PART I)

SYNTHESIS OF DISTRIBUTED SYSTEMS

and

(PART II)

N67 29178

SIGNAL FLOW GRAPH APPROACH TO COMPUTER-AIDED DESIGN

September 1, 1966 - February 28, 1967

Principal Investigator

Gerald H. Cohen

Department of Electrical Engineering
College of Engineering and Applied Science
The University of Rochester
Rochester, New York

(PART I)

SYNTHESIS OF DISTRIBUTED SYSTEMS

A. INTRODUCTION

This investigation is concerned with adapting a modern hybrid computing facility to the synthesis of systems where the active as well as passive elements may be distributed in space or where combinations of lumped devices and distributed transmission systems may exist. We are thus concerned with guided waves in a inhomogeneous and possibly time-varying media. The viewpoint taken is that the distributed parameters (e.g. the resistance and capacitance which can vary continuously as a function of distance in the direction of propagation) are to be optimized so as to achieve a cost or penalty function minimization. Practical boundary and state variable constraints are to be satisfied such as inequality constraints on the values of the distributed parameters. The result is a determination of the optimal electrical properties of the media as a function of position in the direction of wave propagation.

For the case where the net energy absorbed is positive the material will be passive and an optimal interstage or transmission or filter or equalizer will be obtained.

It is conceivable that energy may be pumped into the waveguide or transmission line by means of parametric excitation of the dielectric constant. Examples may be distributed time varying capacitance or inductance as in the ferromagnetic magnetic amplifier. See the sketch below of a transmission line parametric amplifier equivalent circuit

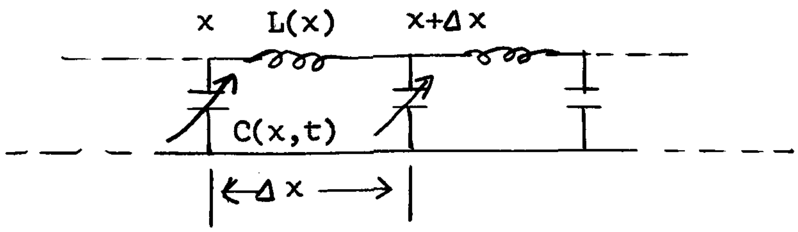


Figure 1

The direct method of the calculus of variations in the form of steepest descent (or the method of gradients) is used to evaluate the optimal distributed parameters. A Hybrid computer set-up for solving a particular example has been determined. A brief description is given in the sequel and the fundamental approach is divulged.

B. GUIDED WAVES

The position taken here is that in integrated circuits at high frequencies waves are propagated or guided from one point to another and it is our task to control the electrical properties of this distributed medium so as to satisfy a certain criteria. We start by developing in brief the generalized transmission line equation for guided waves.

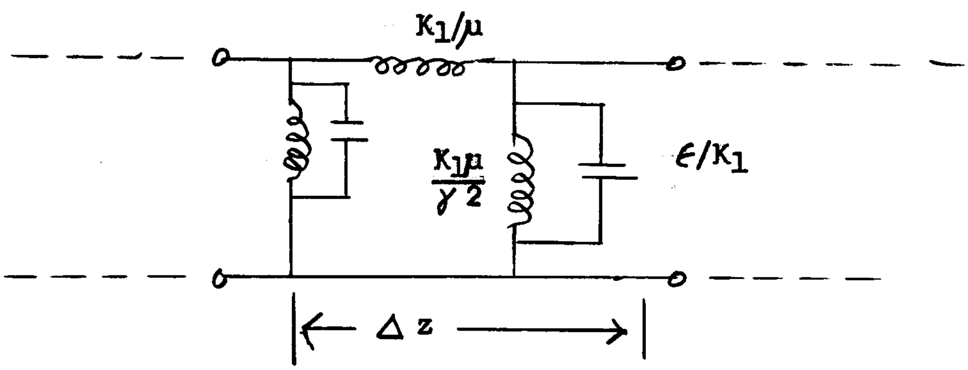
Assume the medium can vary in one direction only - the direction of propagation z, then Maxwell's equation can be split into longitudinal and transverse components, i.e. the gradient can be written $\nabla = \nabla t + \nabla e$ and the electric field intensity E can be written $\bar{E} = \bar{E}_e + \bar{E}_t$ and so on. In this manner we can decompose the waves into their various modes. The results are

$$\frac{\partial}{\partial z} E_e = k_1 \frac{\partial}{\partial t} \mu H_e$$

$$\frac{\partial}{\partial t} \left(\mu \frac{\partial}{\partial z} H_e\right) = \frac{1}{k_1} \frac{\partial}{\partial t} \left(\mu \frac{\partial}{\partial t} \epsilon E_e\right) + \frac{\gamma^2}{k_1} E_e$$

All these quantities E_e, H_e, ϵ, μ are both time and space dependent. If we assume periodic time dependence of E and μ the analysis will lead to a travelling wave parametric amplifier.

For the E mode a general model can be derived. The results are shown below:



Synthesis of an R-C line

The time domain design problem for a series R-shunt C distributed network is one of many that can be handled. For this problem the quantities used are defined in Fig. 1.

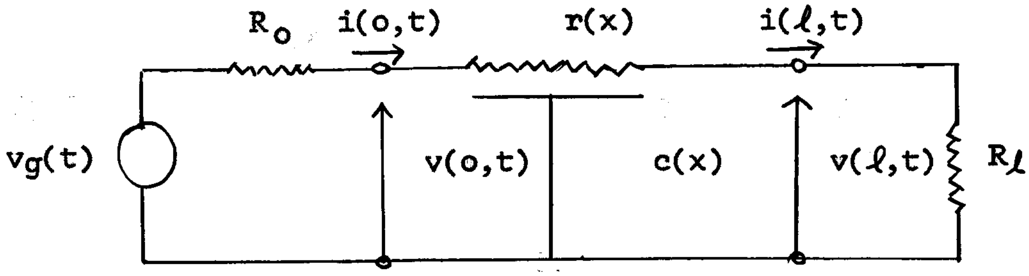


Fig. 1

The statement of this problem is:

Given the source voltage $V_{g(t)}$, source resistance R_o and the load resistance R , find the R and C distributions such that the output voltage $V_{(\ell,t)}$ is the best approximation in the integral square sense to the desired output voltage, over the time interval (o,T).

The system equations are:

$$\frac{\partial}{\partial x} v(x,t) + r(x) i(x,t) = 0$$
$$\frac{\partial}{\partial x} i(x,t) + c(x)\frac{\partial}{\partial t} v(x,t) = 0 \qquad (1)$$

with boundary conditions

$$\gamma_o = v(o,t) + R_o i(o,t) - v_g(t) = 0$$

and
$$\gamma_\ell = v(\ell,t) - R\, i(\ell,t) = 0 \qquad (2)$$
$$v(x,o) = i(x,o) = 0$$

$v_d(t)$ is the desired output voltage

Our task is to find r(x) and c(x) such that

$$\phi = \int_o^T \left[v(\ell,t) - v_d(t)\right]^2 dt \qquad (3)$$

is minimized subject to the physical inequality constraints

$$R_m \leq r(x) \leq R_M \qquad (4)$$

and
$$C_m \leq c(x) \leq C_M$$

Rohrer[1], Resh and Hoyt have attempted a solution to this problem by applying the calculus of variations. The variations in the neighborhood of the optimal distributions yield a set of integro-partial differential equations as a set of necessary conditions. This is further complicated by being a two-point boundary value problem.

As it will be apparent in what follows, the approach presented here follows a relaxation method of step-by-step improvement converging towards the optimal distribution.

Discretization[2]:

It is convenient to discretize the partial differential equations for the voltage and current. This yields a set of difference-differential equations with variable coefficients. This set is compactly written in vector notation as

$$\dot{\overline{Z}} = A\,\overline{Z} \qquad \text{or} \qquad \frac{d}{dx} z_k = f_k(r,c,\overline{Z}) \tag{5}$$

$$\overline{Z}' = [\overline{V}',\overline{I}'],$$

where $\overline{V}' = [v_o, v_1, \ldots\ldots\ldots\ldots, v_N]$,

and where $\overline{I}' = [i_o, i_1, \ldots\ldots\ldots\ldots, i_N]$,

with $v_n(x) = v(x,\Delta n)$,

and $i_n(x) = i(x,\Delta n)$; where Δ is the discretizing step in time.

$$A = \begin{bmatrix} 0 & R \\ C & 0 \end{bmatrix}$$

where $R = -r(x)\,\mathbb{I}$ $\mathbb{I}$ is an identity matrix of rank (N+1)

$C = -c(x)\frac{1}{\Delta}K$ K is a truncated triangular matrix of rank (N+1)

The boundary conditions are

$$\overline{\gamma}_o[\overline{V}(o),\overline{I}(o)] = 0$$

$$\overline{\gamma}_\ell[\overline{V}(\ell),\overline{I}(\ell)] = 0 \tag{6}$$

and the cost function becomes

$$\phi = \tfrac{1}{2}\Delta \|\overline{V}(\ell) - \overline{V}_d\|^2 \tag{7}$$

Pontryagin's Maximum Principle

Defining a new set of state variables $\overline{Q}(x)$ such that

$$Q_k^2(x) = (v_k - v_{dk})^2 \,; \qquad k = o,1,\ldots\ldots\ldots,N.$$

so that $\dot{\overline{Q}} = \overline{f}_Q(\overline{V},\overline{I})$

We arrive at the Hamiltonian

$$H = \overline{p}_V' R\,\overline{I} + \overline{p}_i' C\,\overline{V} + \overline{p}_Q' \overline{f}_Q$$

where $\overline{p}_V$, $\overline{p}_i$, $\overline{p}_Q$ are the adjoint variables

In the expanded version

$$H = r\left[p_{vo}i_o + \Delta(v_o - v_{do})i_o + \cdots\cdots\cdots \right.$$
$$\left. p_{vN}i_N + \Delta(v_N - v_{dN})i_N\right]$$
$$+ \frac{c}{\Delta}\left[p_{io}v_o + p_{i1}(v_1 - v_o) + \cdots\cdots\cdots \right.$$
$$\left. p_{iN}(v_N - v_{N-1})\right]$$
$$= rP_r + cP_c$$

Maximizing H is a necessary condition for minimizing F. **The direct method is not concerned** with these conditions, but this formulation is used to obtain information about the nature of the solution. We see that we have a bang bang control except for possibly the regions of x, over which $\frac{\partial H}{\partial r}$ or $\frac{\partial H}{\partial c}$ identically vanishes.

Gradient Technique[3]:

The general philosophy behind this technique is to obtain the variation in the criterion functional as a function of the variations in the continuous functions r(x) and c(x). (We will represent the control variables as $\bar{u}$, where $\bar{u}' = [r(x), c(x)]$). The method traces a path in $\bar{u}$ space that uniformly reduces F, thus converging to a local optimum of $\bar{u}$.

Equations (5), (6) and (7) give a complete statement of the problem. The modified criterion function F which incorporates the system equations is given by

$$F(\bar{Z},\bar{u},\bar{\lambda}) = \phi + \bar{\eta}_o'\bar{\gamma}_o + \bar{\eta}_\ell'\bar{\gamma}_\ell + \int_o^\ell \bar{\lambda}'[A\bar{Z} - \dot{\bar{Z}}]dx \qquad (8)$$

$\bar{\lambda}' = [\lambda_o, \lambda_1, \quad \lambda_N]$ is a set of adjoint variables which may be identified as the Lagrange multipliers (functions of x). The quantities $\bar{\eta}_o$ and $\bar{\eta}_\ell$ are constant Lagrange multipliers and the Hamiltonian $H = \bar{\lambda}' A \bar{Z}$.

The first order variation in F is obtained as

$$\begin{aligned} \delta F = {} & \phi_{z_\ell} \delta \bar{Z}_\ell + \bar{\eta}_o' \bar{\gamma}_{oz_o} \delta \bar{Z}_o + \bar{\eta}_\ell' \bar{\gamma}_{\ell z_\ell} \delta \bar{Z}_\ell \\ & + \int_o^\ell [(H_z + \dot{\bar{\lambda}}) \delta Z + (H_\lambda - \dot{\bar{Z}}) \delta \bar{\lambda}] \, dx \\ & + \int_o^\ell H_u \, \delta \bar{u} \, dx - [\bar{\lambda}' \delta \bar{Z}]_o^\ell \end{aligned} \tag{9}$$

For steepest descent one has to choose $\delta \bar{u}$ so that $|\delta \bar{F}|$ is maximized, and $\delta \bar{F}$ is to be negative. In order to avoid a singular problem, we may choose $\frac{1}{2} \int_o^\ell \delta \bar{u}' W \delta \bar{u} \, dx$ as a penalizing function and add it to the right hand side of the equation (9).

Now minimizing δF w.r.t. $(\delta \bar{Z}_o, \delta \bar{Z}, \delta \bar{Z}, \delta \bar{\lambda}, \delta \bar{u})$ we obtain $H_\lambda' - \dot{\bar{Z}} = 0$ which are the system equations (5) and $H_z + \dot{\bar{\lambda}} = 0$ which are the adjoint equations

$$\dot{\bar{\lambda}} = - A' \bar{\lambda} \tag{10}$$

$$\bar{\lambda}_o + \bar{\gamma}'_{oz_o} \bar{\eta}_o = 0 \tag{11a}$$

$$\bar{\lambda}_\ell - \phi_z - \bar{\gamma}'_{\ell z_\ell} \bar{\eta}_\ell = 0 \tag{11b}$$

$$\delta \bar{u} = - W^{-1} [H_u'] \tag{12}$$

Equations (11) are the set of boundary conditions for the adjoint variables.

Equation (12) gives the desired variation in u that improves the criterion function - to the first order. Thus, if one starts with an arbitrary nominal control $\bar{u}$ and keeps on updating it in accordance with equation (12) the convergence is obtained when $H_u = 0$ which, however, happens to be a necessary condition for the minima.

Computations:

It is required to solve two sets of differential equations at every stage, both of which are two-point boundary value problems [equations (5), (6), (10), (11)].

A closer look reveals that the system equations are of the form

$$\frac{d}{dx} v_k = - r i_k$$
$$\frac{d}{dx} i_k = - \frac{c}{\Delta} (v_k - v_{k-1})$$

and the boundary conditions

$$v_k(o) + R_o\, i_k(o) - v_{gk} = 0$$
$$v_k(\ell) - R_\ell\, i_k(\ell) = 0$$

The adjoint equations have the same form. This makes it possible to solve the set, pairwise, with the help of a hybrid computer. The analog computer is programmed to find the solution to the two-point boundary value problem by iterating. The digital computer feeds the variable parameters r and c through a D to A converter.

The solutions are stored in the memory of the digital computer which is required to do the algebraic operations and compute $\delta\bar{u}$. The flow diagram is given in fig. 2.

Other Considerations.

(1) Rate constraint: In the case of thin film circuits it seems reasonable to incorporate a constraint on the rate of variations in r and c. This can be done by adding a penalty function of the form $\frac{1}{2}\int_o^\ell \dot{\bar{u}}' P \dot{\bar{u}}\, dx$

where P is a symmetric matrix.

Addition of this penalizing function to equation (8) eliminates the need for an arbitrary penalizing function $\frac{1}{2}\int_o^\ell \delta\bar{u}' W \delta\bar{u}\, dx$

The accessory problem of choosing $\delta\bar{u}$ for minimizing δF has a solution,

$$\delta\bar{u} = P^{-1} {}_0\int^x {}_0\int^\alpha H_u' \, d\beta \, d\alpha + \bar{K}_2 x + \bar{K}_1 - \bar{u} - \bar{u}(o)$$

$$\bar{K}_2 = -\tfrac{1}{2} H_u' x + {}_0\int^x H_u' \, d\alpha$$

$$\text{Sgn } \bar{K}_1 = - \text{Sgn } {}_0\int^1 H_u' \, dx$$

The magnitude of the components of $\bar{K}_1$ should be small enough so that the perturbation equations and first order approximations are valid. The upper limit on the magnitude of the components of $\bar{K}_1$ is obtained from the constraints on $\bar{u}$.

(2) Second Variation[4].

A valid objection may be raised about the rate of convergence and the number of iterations near the optimal solution. The convergence can be improved by taking into consideration the second variation. A formulation based on Bullock's paper[4] was attempted. Obtaining the iterative procedure poses no problem. However, the set of differential equations required to be solved takes us back to the old problem of multivariable state equations with split boundary conditions. The improvement in convergence may not offset the effort involved in solving the equations.

Furthermore, the problem of slow convergence will be faced in the neighborhood of the optimal solution. And since we have to worry about the practical tolerances in synthesizing the network the extra effort in obtaining the optimal control is not warranted.

Conclusions.

The approach adapted here gives successive improvement in the design. The hybrid computation along with the pairwise separation of state and adjoint equations makes it possible to tackle the discouraging task of solving multivariable state equations with split boundary conditions. The rate constraint avoids the singular solution. The gradient technique will also help to solve the problem of singular control.

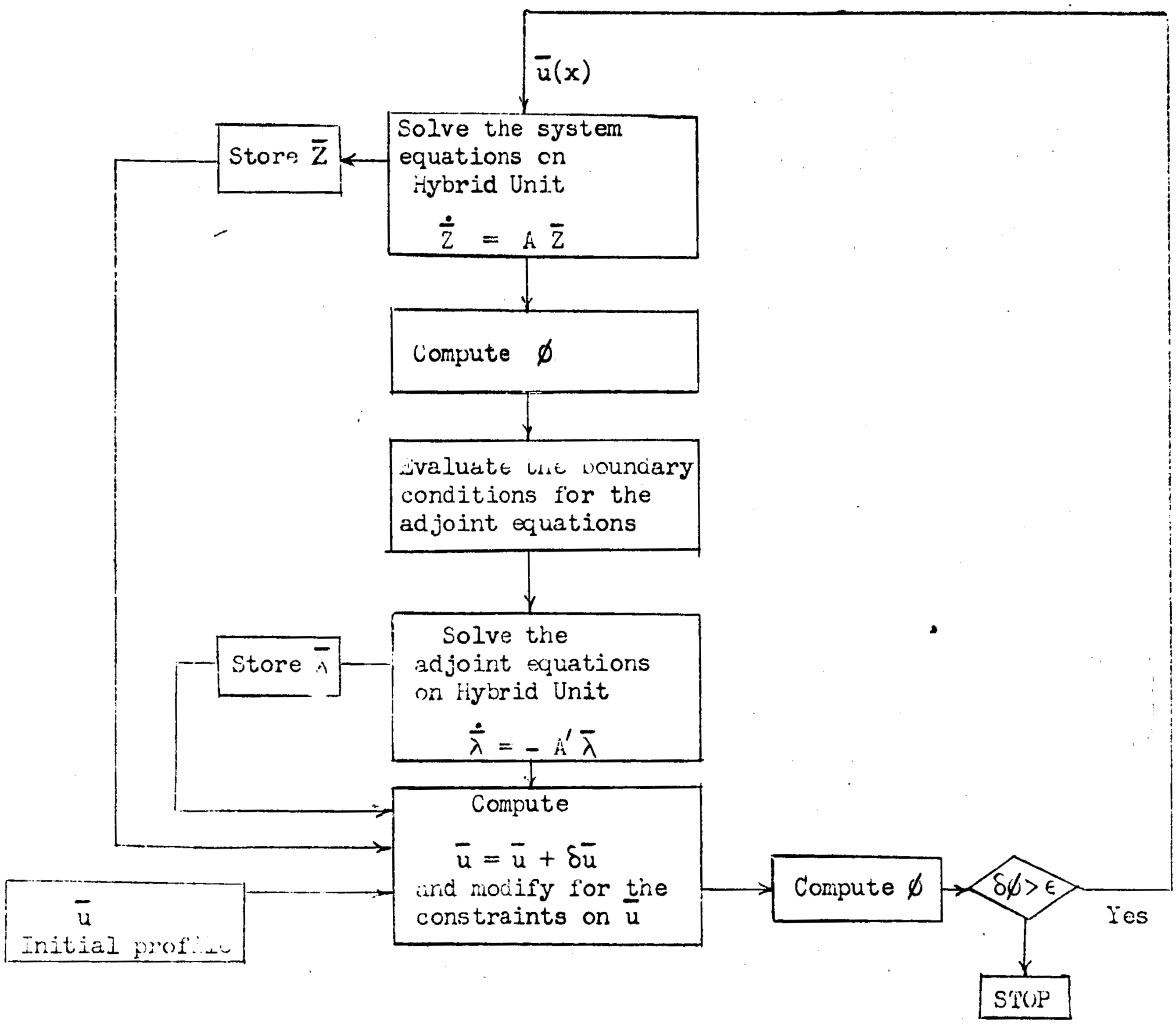


Fig. 2 . Flow Diagram

References.

1. R. A. Rohrer, J. A. Resh and R. A. Hoyt. "Distributed Network Synthesis for a Class of Integrated Circuits". IEEE Convention Record, Vol. 13, pt. 7, 1965, pp. 100-112.

2. E. C. Bertnolli, C. A. Halijak. "Distributed Parameter RC Network Analysis". 1966 IEEE International Convention Record, Part 7, pp. 243-249.

3. A. E. Bryson, W. F. Denham, F. J. Carroll and M. Kinya Mikami. "Determination of Life or Drag Programs to Minimize Re-entry Heating". J. of Aerospace Sciences, April 1962, pp. 420-430.

4. T. E. Bullock. "Optimization and Control Using the Second Variation". Feb. 20-24, 1967, Fourth Winter Institute on Advanced Control, Gainesville, Florida.

Publications Based on this Work.

"Optimal Design of Distributed Parameter Circuits" by A. R. Karnik and G. H. Cohen. To be presented at the 10th Midwest Symposium on Circuit Theory, May 18, 1967, Purdue University, Lafayette, Indiana.

(PART II)

SIGNAL FLOW GRAPH APPROACH TO COMPUTER-AIDED DESIGN

The design procedure using computers can be as shown in the following flow graph:

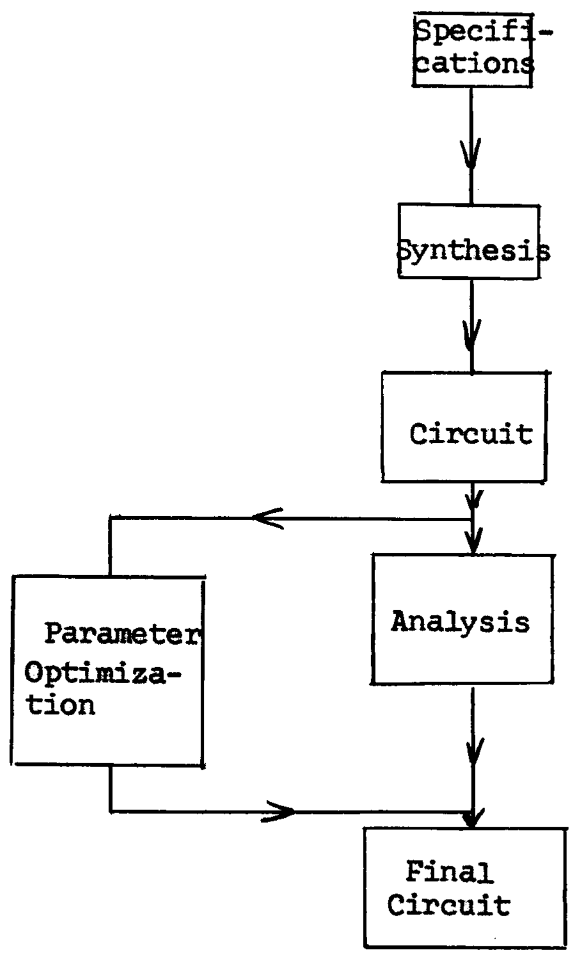


Figure 1 - Circuit Design Flow Chart

We have shown in part I how the synthesis part of the design procedure can be handled using a hybrid computer to solve the variational problem. Next, we will show how the hybrid computer is to carry out the rest of the design procedure.

We have directed our attention in this period to the problem of analysis and simplification. It is assumed that flow graph equations are communicated to the computer via a light pen and CRT.

The method used is based on the Primitive Flow Graph first developed by Mason(1) and also used by Seshu and Balbanian(2) which is related to Bashow's "A" Matrix as are most of the present computer programs(3). In fact, Happ(4) and his NASA group use Mason's method as well as Yang(5).

Our approach to the problem of analysis is based on the observation that the digital computer programs used such as ECAP, SCEPTRE etc. use the digital computer to simulate the network. The analog computer is much more versatile at simulation and when combined with a small scale digital computer (the combination being called Hybrid) analysis as well as parameter optimization may be carried out much less expensively and on line. Thus, all the routines shown in Figure 1 can be carried out on line which means that the human can intervene at will. We feel that this is most desirable. When time-shared digital computers are available and when their use can be shown to be competitive with Hybrid computers then the analog element can give way to the digital and the existing small-scale computer can act as a buffer.

After the digital computer has digested the information presented to it by the CRT display, a program has been written to plot the signal flow graph automatically from the flow graph connection matrix.

A Gauss elimination technique is used to solve for any of the variables. This routine can handle complex numbers as required for a.c. analysis. This allows us to determine the influence any branch has on a variable represented by a node for a particular frequency and driving source. We store both the magnitude and phase. Then each branch value is eliminated one at a time, and new node value is determined. The ratio of the magnitudes of the differences between the magnitudes of the node value with and without the branch is determined. The differences in the phase angle is also determined. If these differences both in magnitude and phase are within a preset rejection criteria, the branch can be eliminated. If not, the branch is retained. Because some transmittances have a greater effect at some frequencies than at other frequencies, it is necessary to retain a branch in the signal flow graph if it did not meet the rejection criteria for all the frequencies of interest. A simplified flow graph is then drawn for the designer's use.

The signal flow graphs for the original graph and for the reduced graphs for two different criteria are shown in figures 2, 3 and 4. The original circuit is shown in figure 5. It is readily observed that the direct path has been eliminated from the input I_B to the emitter voltage V_E. Note also that the current feedback node I_{f2} can also be eliminated. This will be done in our next revision of the program. The same is true for I_{f3}. The branches connected to these nodes will then be eliminated.

This particular example was taken from an article by Waldhauer[6]. Note that our reduced graph will be simpler and that it was obtained rigorously, - not by intuition. If desired, the simplified flow graph at low, medium and high frequencies may be separately drawn to show the influence of the reactive elements.

The signal flow graph is easily converted to an analog computer patch-up by inspection. However, to avoid error, a program will be written giving specific typed-out instructions to patch the analog computer. The simplification procedure can be used to simplify the patching as well as to simplify any digital simulation.

References

1. S. J. Mason and H. J. Zimmerman. "Electronic Circuits, Signals and Systems", Chapter 5. John Wiley and Sons, Inc., 1960, pp. 140-145.

2. S. Seshu and N. Balabanian. "Linear Network Analysis", Chapter 10. John Wiley and Sons, Inc., 1959, pp. 407-425.

3. F. H. Branin, Jr. "Computer Methods of Network Analysis", Seminar: Computer-Aided Circuit Design, M.I.T., Cambridge, Mass., April 11, 1967.

4. W. W. Happ. "Flowgraphs as a Teaching Aid", IEEE Transactions on Education, July 1966, pp. 69-80.

5. T. Yang. "Semi-Annual Report on Computer-Aided Analysis", NASA Grant NGR-39-023-004, November 1966.

6. D. Wildhauer. "Wideband Feedback Amplifiers". IRE Transactions on Circuit Theory, Vol. CT-4, No. 3, September 1957.

Publications Based on this Work

"Computer Generation and Reduction of Signal Flowgraphs". To be published June 1967 as a M.S. Thesis by J. Zigadlo.

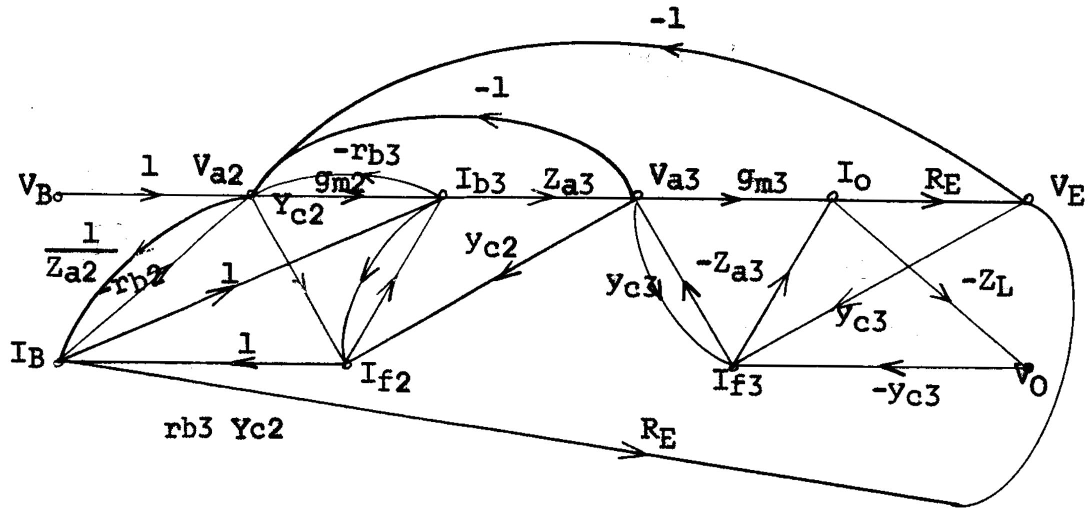


Figure 2 - - Original Signal-flow graph for composite stage.

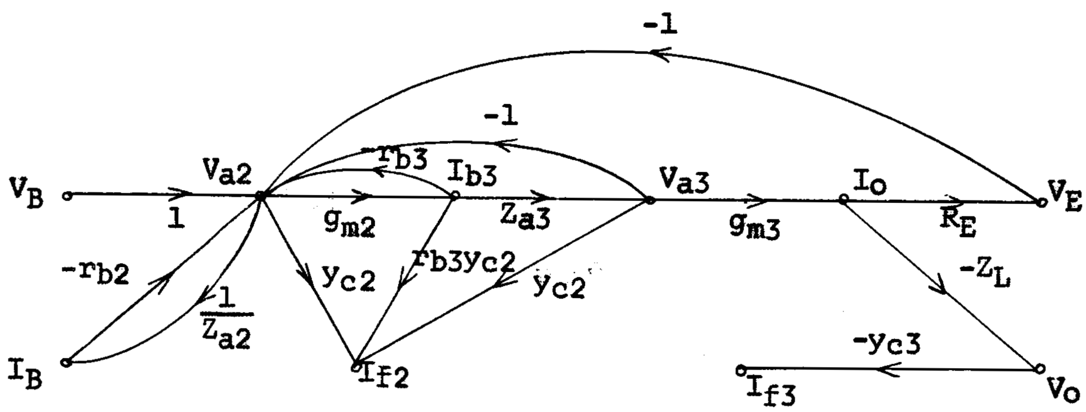


Figure 3 - Reduced Signal Flow Diagram with magnitude criteria 0.9 to 1.1 and phase criteria $\pm$ 6.5°.

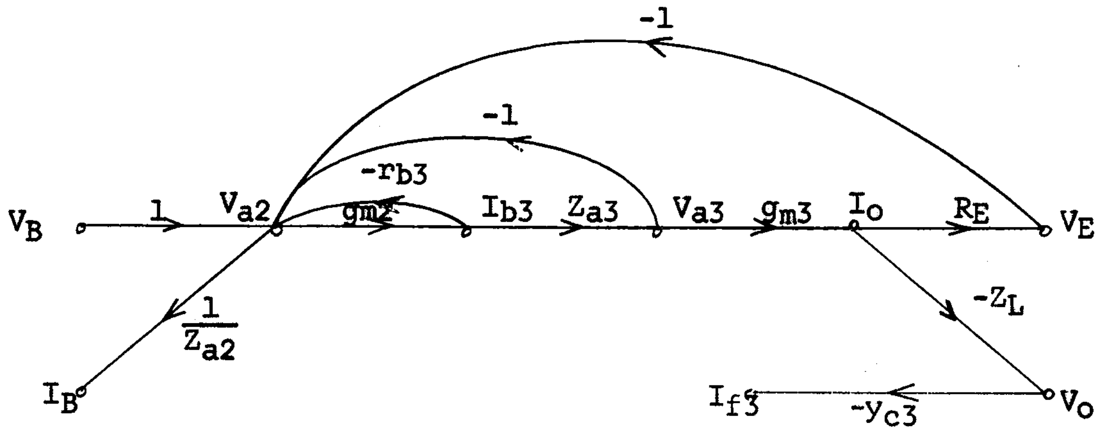


Figure 4 - Reduced Signal Flow Diagram with magnitude criteria 0.65 to 2.5 and phase criteria ± 6.5°.

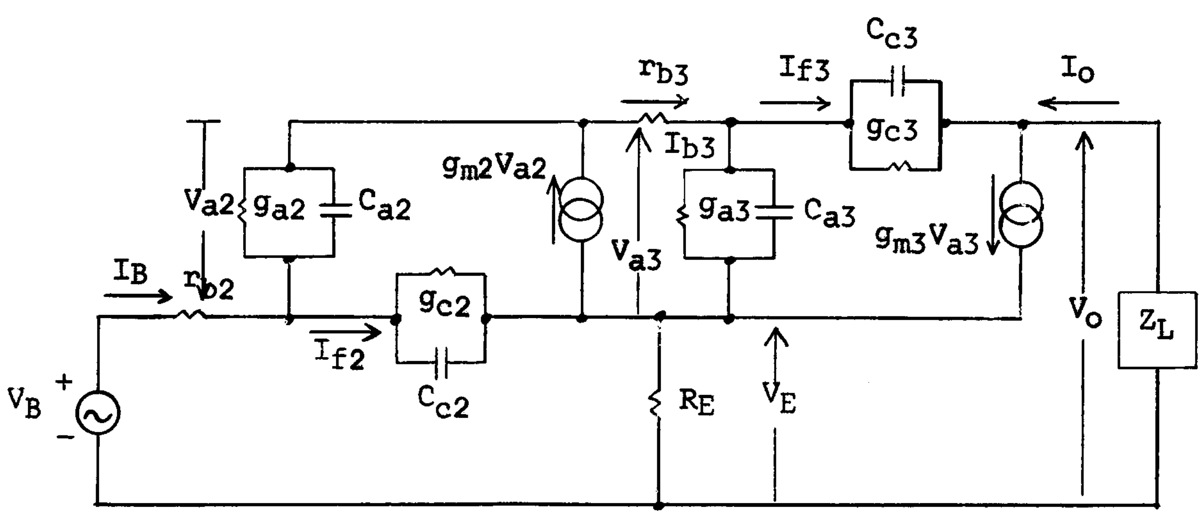


Figure 5 - Equivalent Circuit of the Composite Stage using hybrid-pi transistor parameters.